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ESTIMATION OF OUTPUT GAP IN POLISH ECONOMY USING STRUCTURAL VAR MODELS

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Abstract. The paper presents a method of estimating the output gap for Poland, advised by Blanchard and Quah [1989]. This method stems from the traditional Keynesian and neo-classical synthesis, with identifies potential output with the aggregate supply capacity of the economy and cyclical fluctuations with changes in aggregate demand. There were made an estimation and verification of vector autoregression model VAR(2) for the growth rate of Gross Domestic Product (GDP) and unemployment rate, than of its structuring. Determination of the reaction function of demand and supply, in particular the reaction function of GDP growth on the demand-side disorder, allowed the estimation of the size of output gap. Potential output can be described as following a random walk if production impulse evolve as a stochastic trend, for example, if productivity growth depends on the stochastic arrival of new technologies.

Key words: output gap, structural vector autoregression models, impulse response function, supply and demand shocks

INTRODUCTION

Output gap is defined as a difference between the actual and the potential GDP, based on non-observable production potential. Potential product is circumscribed in the literature as the maximum GDP possible to creation in economy of the country under conditions of full employment, which do not cause inflationary pressures. This unemployment rate is defined as NAIRU (*Non-Accelerating Inflation Rate of Unemployment*). In turn the New-Keynesian Dynamic Stochastic General Equilibrium Models

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(DSGE) with sticky prices [Clarida, Gali and Gertler 2000] indicates the potential product as a possible to achieve in an economy with perfectly flexible prices and wages [Gradzewicz and Kolasa 2004].

Since the dimension of potential GDP is not observable, determination of its size is difficult, often ambiguous. Methods for estimating the output gap based on the definition of Okun [1962] usually use non-linear character of the Cobb-Douglas production function. In turn the New-Keynesian theory assumes that potential GDP is achieved in the economy in the long run in the absence of stiffness. To this refers the approach based on the decomposition of GDP on durable and cyclical component, where potential is identified with long-term growth path.

In this thesis there will be presented a method for estimating the output gap for Poland based on product decomposition (*Permanent-Transitory Decomposition*) by restriction imposed on vector autoregression model, proposed by Blanchard and Quah [1989].

OBJECTIVE AND METHOD OF RESEARCH

The aim of this revision is to present the results of studies concerning the formation of output gap in Polish economy in the years 1996–2011. To estimate the output gap there were used a vector autoregression model (VAR) and its structural form (SVAR).

For *n*-dimensional vector of macroeconomic variables $Y_t = [Y_{1t}, Y_{2t}, ..., Y_{Nt}]'$, which forms the tested structure and between which there is a relationship defined by economic theory, VAR(P) model can be presented [Lütkepohl, 1990] as:

$$Y_t = A_0 + A_1 y_{t-1} + A_2 y_{t-2} + ... + A_P y_{t-P} + \varepsilon_t, \tag{1}$$

where ε_t is *n*-dimensional white noise process with expected value equal to 0 and covariance matrix $\mathbf{\Sigma}$, matrix A_0 contains free words for individual equations, and matrices A_P: $p = 1, 2, ..., P$ represents an impact of P-ties depended variable lag on its current value. Desirable features of VAR(P) model are its stationarity and reversibility. System stationarity can be described as the impact expiration of a ε_t shock on the vector of variables expounded:

$$\lim_{k\to\infty} = \frac{\partial Y_{t+k}}{\partial \varepsilon_t} = 0. \tag{2}$$

The only source of shock in the VAR(P) model is the random components, which makes the condition of stationarity equivalent to the existence of long-term value for Y_t, to which the process returns. The pace of this return is determinate by the roots of the characteristic equation:

$$| A(z) | = 0. \tag{3}$$

Vector autoregression model parameters can by estimated by OLS, separately for each of the equations, which arise from the fact that the explanatory variables are settled in advance variables independent to random component. From the VAR(P) model specification viewpoint it is important to determinate the range of p lags. Since the economic theory does not provide information on the dynamic relationship between variables, to their determination there will be used information criteria: AIC, HQ, SC and FPE [Hamilton 1994]. Guideline in selecting the maximum lag can also be further lags significance test of VAR models. Verification of the null hypothesis: $H_0 : A_P = 0$ is made on the test of quotient of the likelihood function, whose statistics is determined by the formula:

$$LR = T(\ln |\hat{\Sigma}_{re}| - \ln |\hat{\Sigma}_{ur}|), \tag{4}$$

where $\hat{\Sigma}_{re}$ and $\hat{\Sigma}_{ur}$ are covariance matrices for the random components for the model with restrictions and without restrictions. With truthfulness of null hypothesis the LR statistic has asymptotic distribution $\chi(N^2)$, where N^2 is equal to the number of parameters, for which the restriction was imposed to zero. Improper model specification based on too low selection of lags to VAR model may cause the phenomenon of residual autocorrelation, an in further consequence the covariance matrix estimator is loaded. Autocorrelation can be detected [Lütkepohl and Kratzig 2004] using multivariate Breusch-Godfrey test also determined as *adjusted Portmanteau Test*. Covariance matrix for the n-dimensional stationary process Y_t is presented with following formula:

$$\Gamma_S = \operatorname{cov}(Y_t; Y_{t-s}). \tag{5}$$

The estimator of this matrix for the test (attempt) is:

$$\hat{\Gamma}_S = \frac{1}{T}\sum_{t=s+1}^{T} Y_t Y_{t-s}^{'}. \tag{6}$$

Multivariate Ljung-Box test statistics is presented with the following formula:

$$LB = T^2 \sum_{j=1}^{J} \frac{1}{T-j} tr(\hat{\Gamma}_j^{'} \hat{\Gamma}_0^{-1} \hat{\Gamma}_j \hat{\Gamma}_0^{-1}), \tag{7}$$

and with the truthfulness of the null hypothesis it has resolution χ^2 with $N^2(J-P)$ degrees of freedom.

Giving an economic interpretation of VAR models is done by their structuring. Structural vector autoregression model (SVAR) can be written as:

$$Ay_t = C_0 + C_1 y_{t-a} + C_2 y_{t-2} + \ldots + C_P y_{t-P} + B\eta_t, \tag{8}$$

where the random components of the individual equations have normal distribution and are mutually orthogonal $\eta_t \sim N(0; I_N)$. Since the random components are independent relative to each other, they are interpreted economically as a structural shocks, i.e. the demand, supply, monetary or exchange rate shock. The reaction of endogenous variables on stimuli in the form of structural shocks is described by the impulse response function (*impulse-response function*, IRF), and its value after k-periods shows the relationship:

$$IRF_{k(i,j)} = \frac{\partial Y_{i,t+k}}{\partial \eta_{jt}}. \tag{9}$$

As the number of parameters for SVAR model is larger than for VAR model by $N^2 + \frac{N(N-1)}{2}$, in order to obtain a parameters for SVAR model it should be imposed by the exact number of restrictions. Identification conditions are mostly the consequence of the selection of variables for model and economic knowledge about the modeled phenomenon. Because in SVAR model used to calculate an output gap are shocks exerting long-term impact on the selected endogenous variables included in vector Y_t, identification was carried out using the long-term identifying restrictions proposed by Blanchard and Quah [1989]. Restrictions are imposed on matrix ψ, which describes the impact of long-term, cumulative structural shocks η_t on variables of SVAR model [Hamilton 1994]:

$$\Psi = \lim_{k\to\infty} AIRF_k = (I - A_1 - A_2 - \ldots - A_P)^{-1} A^{-1} B. \quad (10)$$

Long-term structuring assumes imposition of $\frac{N(N-1)}{2}$ zero restrictions on matrix $\boldsymbol{\psi}$ and restrictions on matrix **A**, so that **A** = **I** (identity matrix). Restriction $\Psi_{ij} = 0$ means that the impact of acumulated *j*-th structural shock η_{jt} on *i*-th endogenous variable Y_{it} is zero. Blanchard and Quah [1989] found that dynamics of unemployment rate and GDP growth are determined by supply and demand structural shocks, but the long-term impact of demand shock on GDP is zero. Theoretical model justifying the introduction of these restrictions is the system of equations:

$$\begin{aligned} y_t &= m_t - p_t + a\theta_t, \\ y_t &= n_t + \theta_t, \\ p_t &= w_t + \theta_t, \\ w_t &= w \mid E_{t-1} n_t = \overline{n}, \end{aligned} \quad (11)$$

where: y_t – GDP logarithm, n_t – employment logarithm, θ_t – performance logarithm. Full employment is defined as $\overline{n}$, and p_t, w_t and m_t are logarithms accordingly of the level of prices, minimum wage and money supply. The first model equation (11) represents the aggregate demand, which is a function of real money supply and productivity, the second describes the function of supply based on employment and productivity understood as a technology.

Another concerns the pricing mechanism in the economy, which is a resultant of nominal wage and productivity. Wages are determined in the previous period so that it is possible to achieve in the current period the expected full employment. For the analytical solution of system (11) it is assumed that m_t and θ_t are generated by random walk processes:

$$\begin{aligned} m_t &= m_{t-1} + \varepsilon_{Dt}, \\ \theta_t &= \theta_{t-1} + \varepsilon_{St}, \end{aligned} \quad (12)$$

where ε_{Dt} and ε_{St} are uncorrelated in time and orthogonal to each other disorders on the part of demand and supply side accordingly. For the above system there can be also defined an unemployment rate as the deviation from the state of full employment $\overline{n} - n$, which leads to solving the model for product and unemployment growth as a function of supply and demand shocks:

$$\Delta y_t = \varepsilon_{Dt} - \varepsilon_{D(t-1)} + \lambda(\varepsilon_{Dt} - \varepsilon_{D(t-1)}) + \varepsilon_{St},$$
$$u_t = -\varepsilon_{Dt} - \lambda\varepsilon_{St}. \quad (13)$$

In the short term both the demand and supply disruption have impact on the GDP growth, as well as unemployment rate. In the long term model (11) is characterized by a lack of impetus to the demand impact on product growth. Only ε_{St} impacts on long-term on Δy_t. The method proposed by Blanchard and Quah [1989] is an estimation of VAR model, in which the GDP dynamics and unemployment rate is termed by the above-described system in reduced form.

MODEL ESTIMATION, VERIFICATION AND STRUCTURING

There has been analyzed quarterly data concerning GDP growth pace (logarithmic increases) as well as the unemployment rate from I quarter of 1998 to IV quarter of 2011. Figure 1 shows the processes that generate realization of GDP and unemployment rate.

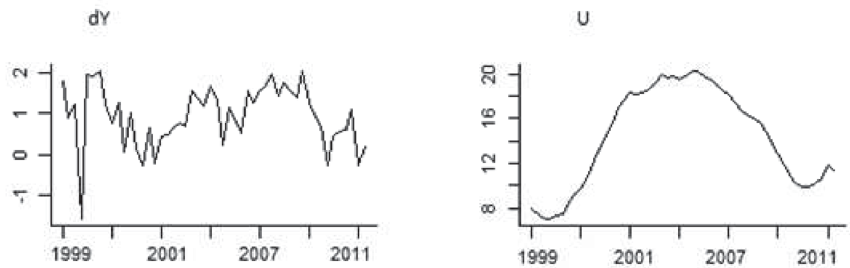

Fig. 1. Chart of time series for the product of GDP and the unemployment rate
Rys. 1. Wykres szeregu czasowego dla PKB oraz stopy bezrobocia

Source: own work
Źródło: opracowanie własne

In the first step, using the information criteria, there have been established a lag range for model VAR(p).

Table 1. Information criteria for VAR model
Tabela 1. Kryteria informacyjne dla modelu VAR

Lag	AIC	HQ	SC	FPE
1	–1.669	–1.580	–1.431	0.188
2	–2.455	–2.306	–2.058	0.085
3	–2.454	–2.245	–1.897	0.086
4	–2.349	–2.081	–1.634	0.096

Source: own work
Źródło: opracowanie własne

All the information criteria clearly show on maximum lag P = 2. A guideline in selecting the maximum lag for described VAR model is also a significant test 3 lag. The likelihood ratio test statistics (4) for VAR(3) equal to LR = 8.37 with the empirical significance level equal to 0.0788 indicates that on significance level of 5% for analysis there should be selected model VAR(2).

Estimation of VAR(2) model was performed in program **R**. Below there is presented a print fragment from the package for the estimation of model parameters.

```
VAR Estimation Results:
Endogenous variables: dY, U              Deterministic   variables:
const
Sample size: 54                         Log Likelihood: -73.882
Roots of the characteristic polynomial:    0.882 0.882 0.547 0.376

Estimation results for equation dY:
dY = dY.l1 + U.l1 + dY.l2 + U.l2 + const
     Estimate   Std. Error           t value    Pr(>|t|)
dY.l1    0.173        0.152      1.14           0.26
U.l1    -0.111        0.167      -0.67          0.51
dY.l2    0.209        0.153      1.37           0.18
U.l2     0.137        0.165      0.83           0.41
const    0.200        0.402      0.50           0.62
Residual standard error: 0.716 on 43 degrees of freedom
Multiple R-Squared: 0.166,      Adjusted R-squared: 0.0881

Estimation results for equation U:
U = dY.l1 + U.l1 + dY.l2 + U.l2 + const
     Estimate   Std. Error      t value    Pr(>|t|)
dY.l1   0.0170        0.0911     0.19           0.853
U.l1    1.7468        0.1000     17.47          < 2e-16
dY.l2  -0.0168        0.0918     -0.18          0.856
U.l2   -0.7759        0.0987     -7.86          7.5e-10
const   0.4441        0.2411     1.84           0.072 .
Residual standard error: 0.43 on 43 degrees of freedom
Multiple R-Squared: 0.991,      Adjusted R-squared: 0.99
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Model specification also requires examination of the phenomenon of residual autocorrelation, which may be a result of too low selection of maximum lag P. The value of the Ljung-Box (*adjusted Portmanteau Test*) test statistics equal to LB = 11.9 with p-value = 0.7497 indicates that in the VAR(2) model on the significance level of 5% the problem of autocorrelation residuals does not occur. From the perspective of predictive ability of VAR(2) system an important aspect is a fact of model stability. For this purpose, there was a CUMSUM statistic graph constructed for the system describing a GDP dynamics and the unemployment rate in Poland (Fig. 2).

Based on the chart in Figure 2 we can assume that the VAR(2) model is stable.

The next step was to build a SVAR model based on the VAR(2) model as well as a Blanchard and Quah assumptions. In two-dimensional structural model of vector autoregression the dynamics of unemployment rate and product growth is associated with supply and demand structural shocks. In order to identify them, on the VAR model there were imposed restrictions, that the long-term impact of the demand disturbances on GDP is zero. SVAR model was estimated also using the **R** package [Kleiber and Zeileis 2008].

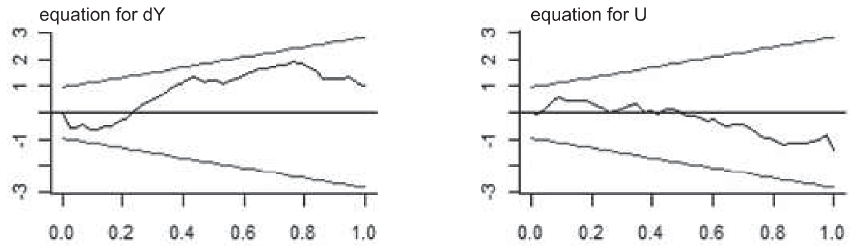

Fig. 2. Chart of CUMSUM statistics for the GDP product and the unemployment rate
Rys. 2. Wykres statystyk CUMSUM dla PKB oraz stopy bezrobocia

Source: own work
Źródło: opracowanie własne

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SVAR Estimation Results:
Estimated contemporaneous impact matrix B:
      dY              U
dY    0.625           -0.348
U     0.156           0.400

Estimated identified long run impact matrix ψ:
      dY              U
dY    1.23            0.0
U     5.36            13.7
```

Supply shock causes an immediate increase of GDP of 0.625% and unemployment rate of 0.156% points. Occurrence of the demand disturbance leads to a fall of GDP by 0.384% and an increase of unemployment rate of 0.4% points. Structuring results also indicates that long-term impact of supply disturbance on GDP is 1.23%, and a demand shock 0 – according to the imposed restrictions. The economic interpretation is not given on long-term impact of shocks on the unemployment rate. For presenting the dynamics of change of GDP growth and the unemployment rate in response to structural shocks there were used an impulse response functions, which are shown in Figure 3 and Figure 4. A positive supply shock affects the growth of GDP while increase of unemployment rate.
As a result of a negative demand shock according to economy theory there comes after a domestic product decrease and an increase of unemployment (Fig. 4).

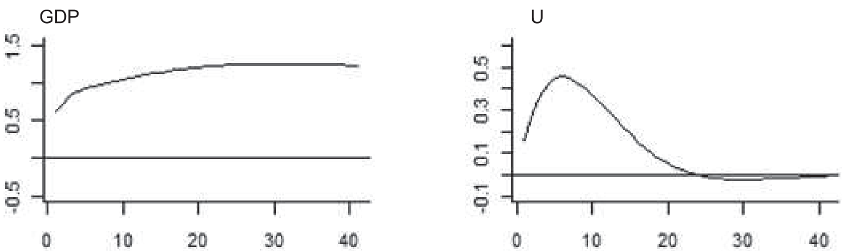

Fig. 3. Chart of impulse response functions on supply impulse
Rys. 3. Wykres funkcji reakcji na impuls dla szoku podażowego

Source: own work
Źródło: opracowanie własne

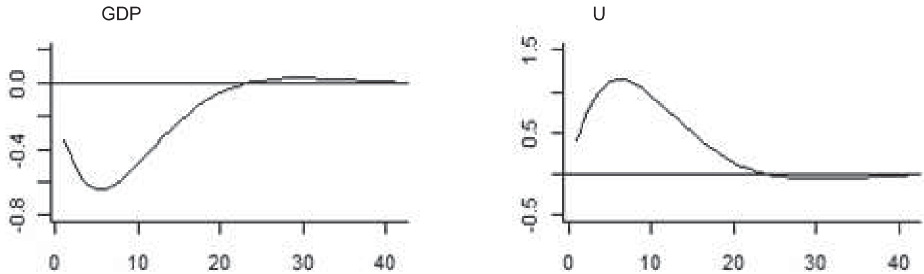

Fig. 4. Chart of impulse response functions on demand impulse
Rys. 4. Wykres funkcji reakcji na impuls dla szoku popytowego
Source: own work
Źródło: opracowanie własne

An output gap can be defined as a part of GDP that results from the presence of past and current demand-side disturbances. In turn a potential product achievable in the economy is the sum of deterministic trend and supply shocks that impact on the GDP. In order to determinate an output gap there have been set VAR(2) model rests, which were used to calculate the realization of structural shocks ($\hat{\eta}_t = B^{-1}e_t$). Since the reaction function of GDP growth on the demand impulse η_t^D is equal to IRF_k^D, then for each period it is possible to determinate the impact of current and past demand shocks $\hat{\eta}_t^D$ on GDP growth, which is equal to

$$\Delta y_{t(D)} = \sum_{k=0}^{h} IRF_k^D \hat{\eta}_{t-k}^D .$$

The value of output gap can be finally defined as:

$$y_t^{GAP} = \sum_{i=1}^{t} \Delta y_{t(D)}. \qquad (14)$$

Figure 5 shows the estimates of output gap for the Polish economy on the basis of a VAR(2) model.

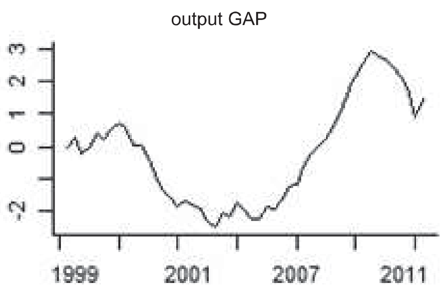

Fig. 5. Chart of estimates of the output gap for the Polish economy
Rys. 5. Wykres luki popytowej dla polskiej gospodarki
Source: own work
Źródło: opracowanie własne

It should be noted that the result is consistent with the economic knowledge of economic processes in Poland. Analyzing chart in Figure 5 is should be noted that from 1998 to IV quarter of 1999 output gap remained close to zero, which indicates that the economy during this period grew at the level of its potential. Since the III quarter of 2000 an output gap fell well below zero level to achieve an amount of around 2% below the GDP potential from I quarter of 2002. From II quarter of 2005 level of the output gap had a positive trend to reach the level of 3% of potential GDP in III quarter of 2008. In 2010 the output gap gape decreased to 1%, but in the second quarter of 2011 it began to grow again.

SUMMARY

This paper has presented new estimates of trend output and the output gap for Poland according to vector autoregression model advised by Blanchard and Quah [1989]. The result was to estimate VAR(2) model and its structural form, which allowed calculation of the level of output gap. A positive supply shock affects the growth of GDP (an immediate increase of 0.625%) while increase of unemployment rate and as a result of a negative demand shock according to economy theory there comes after a domestic product decrease and an increase of unemployment. Its course shown in Figure 5 corresponds to the behavior of real economy and investments processes in the 90's and the beginning of the new century. This paper is a first step to review a different methods (the Hodric-Prescott filter, the Beveridge-Nelson decomposition, system of potential output and the NAIRU) that can be used to estimate potential output and output gap, what is useful to help identify the scope of sustainable noninflationary growth to and to allow an assessment of the stance of macroeconomic policies.

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ESTYMACJA LUKI POPYTOWEJ W POLSKIEJ GOSPODARCE Z WYKORZYSTANIEM MODELU WEKTOROWEJ AUTOREGRESJI (VAR)

Streszczenie. W pracy przedstawiono metodę szacowania luki popytowej dla Polski, zaprezentowaną przez Blancharda i Quah [1989]. Metoda ta wywodzi się z tradycyjnej syntezy keynesizmu oraz syntezy neoklasycznej, identyfikuje potencjalny poziom produkcji z łączną zdolnością podażowa gospodarki i wahań cyklicznych ze zmianami zagregowanego popytu. W celu oszacowania luki popytowej przeprowadzono estymację i weryfikację modelu wektorowej autoregresji VAR(2) dla tempa wzrostu Produktu Krajowego Brutto (PKB) i stopy bezrobocia oraz jego strukturalizację. Określenie funkcji reakcji popytu i podaży, w szczególności funkcji reakcji wzrostu PKB w wyniku zaburzeń popytu, pozwoliły oszacować wielkość luki popytowej. Potencjalną produkcję można opisać jako następstwo błądzenia losowego, jeśli impuls produkcyjny rozwija się jako stochastyczny trend, na przykład jeśli tempo wzrostu wydajności zależy od stochastycznego przyrostu nowych technologii.

Słowa kluczowe: luka popytowa, strukturalne modele wektorowej autoregresji, funkcja reakcji na impuls, szok popytowy i podażowy

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