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APPLICATION OF DYNAMIC MODELS WITH STOCHASTIC PARAMETERS OF THE OBJECTIVE FUNCTION TO THE OPTIMIZATION OF PRODUCTION IN FARMS OF THE WEST POMERANIAN VOIVODSHIP

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Abstract. Two dynamic models of stochastic programming were constructed for the West Pomeranian Voivodship. They comprised four successive years of varied agriclimatic conditions, with and without EU subsidies. Model I referred to farms growing crops, whereas Model II referred to those growing crops and livestock. Both models accounted for random parameters of the objective function, which constituted the matrix of farm income achieved in 10 regions of the West Pomeranian Voivodship. Those models were solved by means of three methods using the MATLAB software. The results comprise a precise area of the particular cultures and the fallows, the joint farm income achieved in the four analyzed years and the risk accompanying the accomplishment of it.

Key words: dynamic model, stochastic programming, farm income, risk

INTRODUCTION

Agricultural production, the crops in particular, is closely related to the natural environment of a random character. This, aside of the agrotechnical treatments, determines the size and fluctuations of yields. The optimization of agricultural production in uncertainty conditions is enabled by stochastic programming models. One of the forerunners that classified the problems of stochastic programming was Schneeweiss [1962]. In his distribution he distinguished models with a random objective function. In his paper [1991] Krawiec attempted to solve such models. They concerned the optimization of farm production in a particular year. The operation of farms over a period of a few years was studied by Jeleniewska [1993]. All parameters in those models were deterministic.

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In the global literature one can find many authors discussing the effect of random factors on selected agricultural problems. For instance, Vercammen [2003] investigated the direct subsidies and payments on farm investments. Pihamaa and Pietola [2002] developed a plan of optimum cattle breeding in uncertainty conditions.

This paper discussed the optimization of the total production of farms in the West Pomeranian Voivodship. Its purpose is to determine the production structure yielding the highest farm income in given conditions while minimizing the risk of its achievement. The risk is related to the achievement of farm income in 10 regions of the voivodship of varied agricultural usability in the 2003–2006 period.

RESEARCH METHOD

The research method comprises dynamic stochastic programming models. The dynamic models consist of blocks created by classical linear programming models. The blocks were connected using balance constraints, hereinafter referred to binding constraints. The conditions were build in accordance with the Bellmann's [Bellman and Dreyfus 1967] recurrence equations principle.

The internal constraints for each block can be written as a matrix form [Grabowski 1980]:

$Ax \leq b$ restrictive (balance) constraints (1)

$x \geq 0$ boundary constraint (2)

where: A – technical-economic parameters;
b – free term;
x – decisive variables.

The objective function of this model has the following form:

$$Z = \boldsymbol{C}^T\boldsymbol{x} \rightarrow max \quad (3)$$

where: $\boldsymbol{C}$ – the vector of random variables with expected value $E(\boldsymbol{C}_i) = c_i$, variance $D^2(\boldsymbol{C}_i) = s^2$ for $i = 1, 2, \ldots, n$ and covariance $cov\ (\boldsymbol{C}_i\boldsymbol{C}_j) = s_{ij}$ for $j = 1, 2, \ldots, m$.

As the objective function provides random variables, we deal with a stochastic programming model. There is a number of ways to solve such a model. This paper uses three methods referred to as the E, V and VE stochastic programming models [Madansky 1963, Krawiec 1991].

If the expected value of the $\boldsymbol{C}$ vector is presented as $E(\boldsymbol{C}) = c = (c_1, c_2, \ldots, c_n)$, then the objective function (3) can be expressed as:

$$E(Z) = E(\boldsymbol{c}^T\boldsymbol{x}) = \boldsymbol{c}^T\boldsymbol{x} \rightarrow max \tag{4}$$

The model established by the formulas (1)–(2) and (4) constitutes a deterministic linear programming problem. Its solution is a vector of the decision variables x_E and the maximum quality z_E which can be achieved in given conditions. This model is referred to as the E stochastic programming model, and the variance δ^2 of its objective function is determined by the formula:

$$x_E^T S x_E = \delta_E^2 \tag{5}$$

where: S – the matrix of variance and covariance of the objective function parameters.

This variance is a measure of the risk, which might appear too high for the decision-maker.

The V stochastic programming model assumes the variance of Z random variable as the objective function which should be minimized. It has the following form:

$$D^2(Z) = \boldsymbol{x}^T\boldsymbol{S}\boldsymbol{x} \rightarrow min \tag{6}$$

This function is a quadratic form, therefore the solution of the model created by the constraints (1)–(2) and the function (6) is possible solely by the application of the quadratic programming algorithm. The solution of the V model is a vector of the decisive variables x_V and the lowest variance δ_V^2 that can be achieved at the given constraints. The expected value is obtained from the formula:

$$z_V = \boldsymbol{c}^T\boldsymbol{x}_V \tag{7}$$

This value is subject to an insignificant risk, however it might be too low for the decision-maker.

If an additional constraint on the expected value of the objective function in the following form is imposed:

$$\boldsymbol{c}^T\boldsymbol{x} \geq d_i \tag{8}$$

where: d_i – any value the interval $[z_V, z_E]$, or from the interval of the ends determined by the expected value in the V and E models, on the V model. Then such a model, created by the balance constraints (1)–(2), (8) and the objective function (6), will be called the VE stochastic programming model. This model will be solved by means of the quadratic programming and allow the choice of the expected quality z_{VE} (calculated by means of the formula 7), which is profitable for the decision-maker, encumbered by an acceptable risk δ_{VE}^2.

CONSTRUCTION OF A FARM MODEL WITH A RANDOM OBJECTIVE FUNCTION

Information upon farm area (the area of arable grounds and green cultures), the crops structure, the livestock capita, the purchase of means of production and services, the sale of crops and livestock products in the 2003–2006 period were collected from databases of the Central Statistical Office, the Agency for Restructuring and Modernization of Agriculture and the Western Pomeranian Branch of Agricultural Advisory Center. The data from particular years constituted technical-economic parameters and free terms of the respective blocks of a dynamic model of an average farm in the West Pomeranian Voivodship. Those blocks were linked to each other by means of balance constraints (mutual), referring to crop rotation and stock turnover. The objective function was created by the matrix of variance and covariance of farm incomes achieved from each activity in the analyzed years in 10 regions of the West Pomeranian Voivodship. The division of the voivodship into regions of similar agricultural usefulness was conducted by means of a discrimination analysis [Zaród 2009]. The unit income for the particular variables of the crop and stock production was calculated as a result between the production value and direct costs (sowing material, fertilizers, pesticides) and other costs (costs of growing and harvesting, others – e.g. twine, foil, taxes, insurance) without the price of the farmer's own work. The assumptions concerning the unit production values and the costs of particular agricultural activities were based on the Western Pomeranian Branch of Agricultural Advisory Center studies [Kalkulacje rolnicze 2003, 2004, 2005, 2006]. Additionally, farm income in the years 2004 and 2005 were increased by direct and supplementary subsidies, whereas in 2006 also by sugar subsidies. The design of such a model is presented in Table 1 [Zaród 2008].

In the analyzed years approximately 84% of farms in the West Pomeranian Voivodship dealt solely with the crop production; every sixth farm kept livestock. Due to the production character two optimization models were developed. Model I, dealing with crop production, consisted of 44 decision variables and 47 constraints. Model II, developed for an average farm dealing with crop and livestock production, comprised 104 variables and 122 balance constraints. Decision variables for each block of Model I describe the crop area of all cereals, bulb and root plants, papilionaceous plants, rape, the purchase of means of production and the rent of labour. Constraints apply to the area of arable land, the crop structure and labour balance. Additionally, binding constraints apply to the area of crops following each other on a particular field (crop rotation), i.e. crop i in year t will be followed by crop j in year t+1. In models with livestock production the decision variables, apart from the abovementioned ones, pertain to the population of animals of relevant classes and species as well as to the purchase of fodder. The balancing conditions were supplemented with livestock site, organic fertilization and fodder demand balances. Furthermore, subsequent years were linked by livestock population changes, for class i animal in year t will move to class j in year t+1. The optimization criterion, depending on the solution method, is the maximization of farm income or the minimization of the objective function value. Square root of

Table 1. Design of linear-dynamic model
Tabela 1. Schemat modelu liniowo-dynamicznego

Constraints	Decisive variables					
	Year I variables	Year II variables	Year III variables	Year IV variables	Sign	Limit
Constraints of years 0–1					≤	
Internal constraints of year 1					≤ ≥	
Constraints of years 1–2					≥	
Internal constraints of year 2					≤ ≥	
Constraints of years 2–3					≥	
Internal constraints of year 3					≤ ≥	
Constraints of years 3–4					≥	
Internal constraints of year 4					≤ ≥	
Multiyear constraints					≤ ≥	
Objective function					→	max (min)

Source: Own elaboration.
Źródło: Opracowanie własne.

variance (formula (6)) is the risk measure. The unit farm income from particular crops and green cultures in all models in the 2004–2006 period is the sum of income from production and subsidies.

RESULTS OF STOCHASTIC PROGRAMMING MODELS SOLUTIONS

The paper used three methods for solving optimization models with random objective function. For Models I and II the E, V and six VE stochastic models were solved (the interval within the ends determined by the qualities of the farm incomes in the models V and E was divided into five parts). The calculations were conducted using the MATLAB software extended with plugins enabling the solution of linear and quadratic programming stochastic optimization models. The results of each model contain the exact area of particular crops and fallows, the total farm income achieved in the four analyzed years and the risk accompanying its achievement. The assumed crop rotation (identical for both models), which ensured maintenance of high culture of soils, had a substantial effect on the results. The area of particular crops and the succession of crops in the analyzed years in the optimal solutions of Model I is presented in Table 2.

Due to a high number of results the table does not account for the solutions of the VE models, dealing with the type and the area of crops.

Table 2. Crop rotation in optimal solutions of Model I
Tabela 2. Zmianowanie roślin w rozwiązaniach optymalnych modelu I

	Model E								
	Field I		Field II		Field III		Field I		Fallow
	ha								
2003	Potatoes Beets Oats	0.30 0.26 2.31	Wheat Barley	1.58 –	Rape Triticale	1.15 0.32	Rye Other crops	2.70 1.74	2.43
2004	Wheat Barley	2.87 –	Rape Triticale	1.27 0.31	Rye Other crops	– 1.47	Potatoes Beets Oats	0.68 – 3.76	2.30
2005	Rape Triticale	1.21 1.66	Rye Other crops	– 1.58	Potatoes Beets Oats	0.3 0.27 0.90	Wheat Barley	3.13 1.31	3.07
2006	Rye Other crops	2.05 0.82	Potatoes Beets Oats	0.41 0.27 0.90	Wheat Barley	1.47 –	Rape Triticale	1.50 2.94	3.43
	Model V								
2003	Potatoes Beets Oats	0.56 – 0.65	Wheat Barley	1.27 –	Rape Triticale	1.16 4.99	Rye Other crops	– 1.74	2.43
2004	Wheat Barley	1.21 –	Rape Triticale	1.27 –	Rye Other crops	4.68 1.47	Potatoes Beets Oats	0.43 0.25 1.06	2.30
2005	Rape Triticale	1.21 –	Rye Other crops	– 1.27	Potatoes Beets Oats	0.61 0.27 5.26	Wheat Barley	1.74 –	3.07
2006	Rye Other crops	– 1.21	Potatoes Beets Oats	0.41 0.27 0.58	Wheat Barley	3.38 2.77	Rape Triticale	1.50 0.24	3.43

Source: Author's calculations using the MATLAB software.
Źródło: Obliczenia własne w programie MATLAB.

All the solutions indicate the most profitable production directions, i.e. wheat and beets. However, the soil requirements of those crops are high and their area in the model was limited to 25 and 2% of the total area of arable land, respectively for wheat and beets. High profit was also ensured by potatoes and rape, however their area was conditioned by crop structure. In optimal solutions, particularly of the V and VE models, there is a tendency to decrease more risky crops in a given year (e.g. rye in 2003, 2005 and 2006, and barley in 2003, 2004 and 2005) and to increase the area of the remaining crops.

The profitability of agricultural production is indicated by the amount of achieved farm income. The farm income (expected value) and the risk of achieving it (standard deviation) jointly in four analyzed years in the optimal solutions are shown in Table 3.

VE models numbered $_{1, 2, \ldots 6}$ are the models, whose target criterion minimises the risk of achievement of farm income. The expected value of farm income in these models falls within the range determined by the V and E models (61,301.95–63,464.34). This range has been divided into six parts corresponding to relevant indexes.

Table 3. Expected value and standard deviation of farm income
Tabela 3. Wartość oczekiwana i odchylenie standardowe dochodu rolniczego

Type of model	Farm income (PLN)	Variance (PLN)	Standard deviation (PLN)
E	63,464.34	22,194,368.99	4,711.09
V = VE_1	61,301.95	9,018,669.67	3,003.11
VE_2	61,734.43	9,308,173.86	3,050.93
VE_3	62,166.91	9,610,062.00	3,100.01
VE_4	62,599.39	9,926,091.32	3,150.57
VE_5	63,031.86	10,280,744.45	3,206.36
VE_6	63,464.34	11,097,159.94	3,331.24

Source: Author's calculations using the MATLAB software.
Źródło: Obliczenia własne w programie MATLAB.

Standard deviation is a measure of risk of accomplishment of the farm income. It increases proportionally to the increase in the farm income. The risk in farming is related primarily to agriclimatic conditions. Year 2003 was disadvantageous for the crops, as was the comparable year 2006. Favorable conditions for crop cultivation ensured high yields in 2004 and slightly lower in 2005. Moreover, the basic and supplementary subsidies for farmers after the Poland's accession to the European Union considerably increased the farm income. The income in the V and VE_1 models, as well as E and VE_6, are identical due to the assumptions of the model. Different risk in the VE_6 model, as compared to the E model, results from the application of a different research method. Conversion of the income to 1 ha of arable land allows for a more precise analysis (Figure 1).

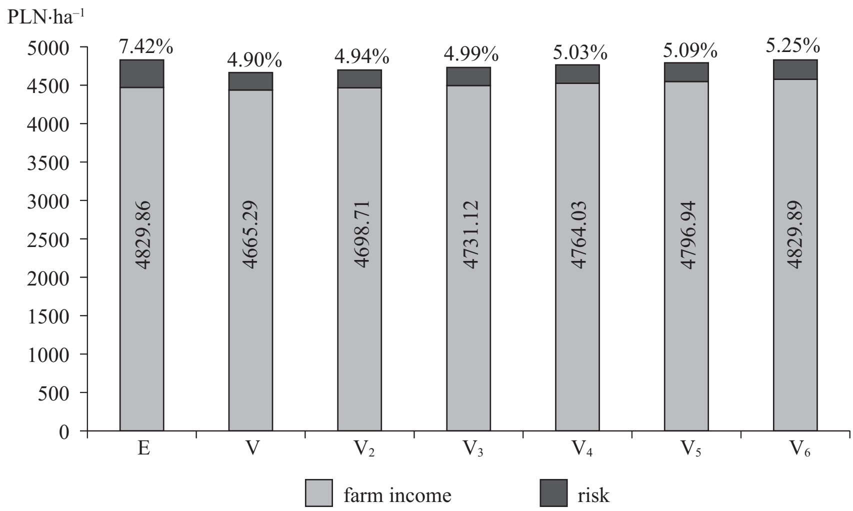

Fig. 1. Unit farm income and the risk of their achievement in Model I
Rys. 1. Jednostkowe dochody rolnicze i ryzyko ich realizacji w modelu I

Source: Own elaboration.
Źródło: Opracowanie własne.

Fluctuation of unit farm income is insignificant, and its highest difference between the E and V models amounts to PLN 164.57. Risk can decrease income by 4.9 to 7.42%. Income reduced by risk is comparable in each solution. The most profitable solution is provided by the VE_6 model, characterized by a high expected value at a considerably low uncertainty of its accomplishment.

The optimal solutions of Model II contain, besides the index of the area of particular crops, the numbers of profitable species of livestock. The analysis of calculations (not all of them can be presented due to technical inconveniences) indicates the profitability of the application of the production structure achieved by means of the VE_6 model. Table 4 contains the area of the crops from this solution.

Table 4. Crop rotation in the optimal solution of Model II
Tabela 4. Zmianowanie roślin w rozwiązaniu optymalnym modelu II

	Model VE_6								
	Field I		Field II		Field III		Field I		Fallow
	ha								
2003	Potatoes Beets Oats	0.38 0.26 0.18	Wheat Barley	1.48 –	Rape Triticale	1.15 4.27	Rye Other crops	0.98 1.66	2.43
2004	Wheat Barley	0.82 –	Rape Triticale	0.99 0.49	Rye Other crops	3.90 1.52	Potatoes Beets Oats	0.38 0.25 2.01	2.30
2005	Rape Triticale	0.15 0.67	Rye Other crops	– 1.48	Potatoes Beets Oats	0.40 0.27 4.75	Wheat Barley	0.84 1.80	3.07
2006	Rye Other crops	– 0.82	Potatoes Beets Oats	0.41 0.27 0.80	Wheat Barley	3.38 2.04	Rape Triticale	1.50 1.14	3.43

Source: Author's calculations using the MATLAB software.
Źródło: Obliczenia własne w programie MATLAB.

The animal production based on a closed turnover of the stock. The optimal solution indicates the profitability of cattle breeding in all analyzed years and the basic livestock consisted of 4 cows, 3.92 calves (calving coefficient equal to 0.98), 3.12 young fatteners, 0.8 replacement heifer and culled cow (5-year-period usability of an adult specimen) each year. The fractions of particular units prove the absence of a given animal on a farm throughout the entire year. Swine breeding was profitable merely in the first two analyzed years. In 2003 the herd consisted of 4 sows, 64 piglets, 62 fatteners, 1 replacement gilt and 1 culled sow. In 2004 the herd decreased to 3 sows, 48 piglets and 62 fatteners while in 2005 merely 46 fatteners remained, reclassified from previous year piglets. The animals were fed with the home fodder and the purchased high-protein mixtures. The commercial production comprised: fatteners, feeder calves, wheat, barley, rape and potatoes (in the case of unprofitability of swine breeding).

The total farm income in the 2003–2006 period in all solutions of Model II was significantly higher than in those of Model I. Its values together with the risk are presented in Table 5.

Table 5. Expected value and standard deviation of farm income
Tabela 5. Wartość oczekiwana i odchylenie standardowe dochodu rolniczego

Type of model	Farm income (PLN)	Variance (PLN)	Standard deviation (PLN)
E	117,832.82	25,135,741.77	5,013.55
$V = VE_1$	105,289.96	10,514,325.06	3,242.58
VE_2	107,798.53	11,616,099.89	3,408.24
VE_3	110,307.10	12,808,453.63	3,578.89
VE_4	112,815.67	14,138,502.41	3,760.12
VE_5	115,324.24	15,809,212.17	3,976.08
VE_6	117,828.48	20,980,980.25	4,580.50

Source: Author's calculations using the MATLAB software.
Źródło: Obliczenia własne w programie MATLAB.

The livestock production increased farm income by 80%. The accomplishment of this income is less risky than in Model I. It may result from a lesser effect of the argiclimatic conditions on livestock breeding. The model assumes a possibility of livestock feeding with the purchased fodder. More precise analysis of income and its risk is presented on Figure 2.

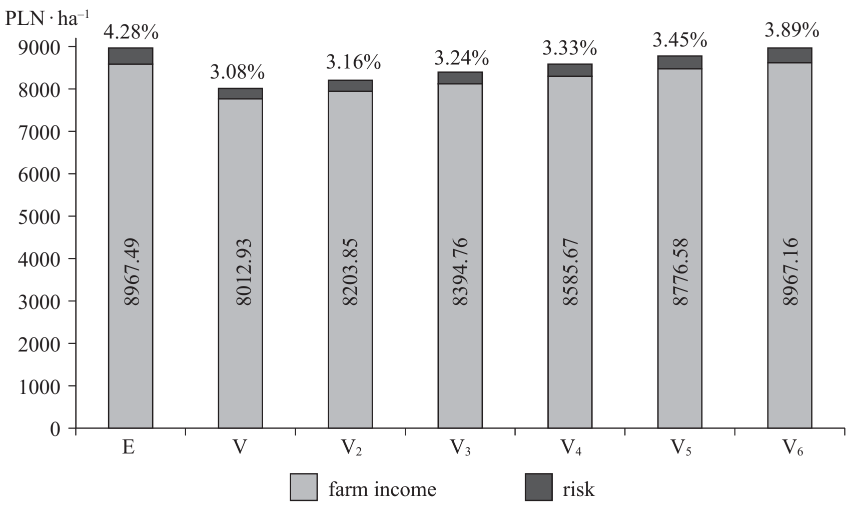

Fig. 2. Unit farm income and the risk of its achievement in Model II
Rys. 2. Jednostkowe dochody rolnicze i ryzyko ich realizacji w modelu II

Source: Own work.
Źródło: Opracowanie własne.

The highest fluctuation of unit farm income for the four analyzed years in Model II amount to 11% (between E and V models). Standard deviation indicates by how much the income might be lower in each solution; the differences fluctuate between 3.08 and 4.28%.

CONCLUSIONS

1. Dynamic models with stochastic parameters of the objective function allow for temporal and spatial alterations. They refer to four successive years (with and without EU subsidies), of varied agriclimatic conditions. Their objective function is constituted by the farm incomes achieved in various regions of the West Pomeranian Voivodship province.
2. Fluctuations of unit farm income, calculated by a few methods, in farms dealing with crop production only are insignificant (amounting to 3.4%). The risk-free income in each solution is comparable.
3. Livestock production increases the income of an average farm in the West Pomeranian Voivodship by 80%, and the risk of its accomplishment does not exceed 4.28% in either solution.
4. The solutions obtained by E models are characterized by high expected value, but also the highest uncertainty of its accomplishment. The V models provide solutions of the lowest quality of the objective function encumbered by an insignificant risk. The application of the VE models (particularly VE_5 and VE_6) determines a structure of production that decreases farm income insignificantly at a relatively low variance.

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ZASTOSOWANIE MODELI DYNAMICZNYCH ZE STOCHASTYCZNYMI PARAMETRAMI FUNKCJI CELU DO OPTYMALIZACJI PRODUKCJI W GOSPODARSTWACH ROLNYCH WOJEWÓDZTWA ZACHODNIOPOMORSKIEGO

Streszczenie. Dla województwa zachodniopomorskiego zbudowano dwa dynamiczne modele programowania stochastycznego. Obejmowały one cztery kolejne lata o różnych warunkach agroklimatycznych, bez i z dotacjami unijnymi. Model I dotyczył gospodarstw zajmujących się produkcją roślinną, a model II uwzględniał uprawę roślin i hodowlę zwierząt. W obu modelach występowały losowe parametry funkcji celu, które stanowiła macierz dochodów rolniczych osiąganych w 10 rejonach województwa zachodniopomorskiego. Modele te rozwiązano trzema metodami za pomocą pakietu MATLAB. Wyniki rozwiązań zawierają dokładną powierzchnię poszczególnych upraw i gruntów ugorowanych, łączny dochód rolniczy osiągany w czterech analizowanych latach oraz ryzyko związane z jego realizacją.

Słowa kluczowe: model dynamiczny, programowanie stochastyczne, dochód rolniczy, ryzyko

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