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APLICATION OF MINKOWSKI'S METRIC IN MEASURING CHANGES OF CONCENTRATION OF VALUE ADDED IN AGRICULTURE, FORESTRY, FISHING AND HUNTING SECTORS

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Abstract. This work is a direct continuation of a previous work by the authors that dealt with construction of new coefficients of concentration by using Minkowski's metric $\rho^p(1 \leq p \leq \infty)$. The following work gives examples of applications of those metrics in agriculture, forestry, fishing and hunting sectors. It also studies the pattern of changes of concentration of added value created in those sectors by comparisons with other sectors.

Key words: differentiation of Polish agriculture, value added, coefficient of concentration, Gini coefficient, Herfindahl-Hirschman Index, Minkowski's metric

INTRODUCTION

During the beginning of the 21st century a series of significant changes in each and every economic sector in Poland. Poland's access to the European Union has enabled obtaining funds for development, especially for nullifying inequalities between regions. After Commission Regulation (EU) 715/2010, dated 10 August 2010 (O.J. EU L 210 dated 11 August 2010), Central Statistical Office published complete series of data from regional accounts in terms of gross national product (GNP) and gross value added (GVA), by types of business activity, according to Polish Classification of Activities – PKD 2007, for the period 2000–2010 [GUS 2012]. That publication allows to study the changes occurring during the period in question, sectioned into 66 subregions. Especially, it allows to partially verify the common hypothesis: EU subsidies diminish regional differentiation. As stated, published data deals with a relatively short period of 2000–2010.

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Naturally, measuring differentiation generally has a multidimensional character and requires relatively large and detailed data sets. The situation is further complicated by migration of people, which is both the cause and effect of changes of value of regional differentiation. Thus, it seems reasonable to begin with a synthetic measure to gauge changes in differentiation in a geographical frame. We can use GVA for a given region as such a synthetic measure that we will use to assess differentiation. A special example of a relatively rarely studied problem is the analysis of changes in agriculture, forestry, fishing and hunting sectors, as those sectors are believed to be "traditional". In the following work we will attempt to answer the question: how is value added changing for those sectors. Also, we will verify whether the changes have the same trends as changes of value added in all sectors combined.

Measurement of concentration is one of key problems in economics. That measurement is done in studies of inequality of income as well as concentration in particular markets. Literature dealing with that problem is abundant. It is worth noting that the most popular tool to study concentration of a market is the Herfindahl-Hirschman Index, denoted sometimes as Herfindahl Index or simply HHI [Herfindahl 1955, Hirschman 1964]. For completeness of our considerations: Calkins [1983], Kwoka [1985], Lijesen [2004], Matsumoto et al. [2012], Djolov [2013]. On the other hand the most commonly utilised coefficient in studies of inequality of income [Barnett 2005] is the Gini Index [Hoffman, Bradley 2007]. However, it must be stated that using only one coefficient, even one that is universally acclaimed, can lead to not noticing occurring changes. Moreover, different coefficients can point to different directions of changes in concentration. That phenomenon will be presented in the next chapter.

MEASUREMENT OF CONCENTRATION

The idea of constructing an indicator that evaluates the phenomenon of concentration is in general based on measuring dissimilarity (differentiation, distance) between a structure of objects and a structure of goods that are owned by the objects. Literature gives many properties that such an indicator should have when it is used to measure concentration. In order to clarify used terms let us assume the following definitions and notations. A set in an n-dimensional Euclidean space $\Re^n$: $\Omega := \{ \mathbf{x} = (x_1,...,x_n) \in [0;\ 1]^n,\ x_1 + x_2 + ... + x_n = 1, x_i \geq 0,\ \mathrm{i} = 1,2,...$, will be denoted as a set of structural vectors or set of structures for short.

Vector $\mathbf{x'} := (x_1^{'}, x_2^{'}, ..., x_n^{'}) \in \Omega$ will be called an ordered structure constructed from structure $\mathbf{x} = (x_1,....,x_n) \in \Omega$, if its coordinates are a permutation of coordinates of vector **x** which satisfies $x_1^{'} \leq x_2^{'} \leq .... \leq x_n^{'}$, which we will denote as $\mathbf{x'} := \mathbf{Px}$ for short, where operator **P** ($\mathbf{P} : [0;1]^n \rightarrow [0;1]^n$) will be called an order operator. Whereas, vector $\mathbf{x^\wedge} := (\hat{x_1}, \hat{x_2}, ..., \hat{x_n})$, where $\hat{x_i} := \sum_{j=1}^{i} x_j^{'},\quad i = 1, ..., n$ will be called a cumulated structure constructed from structure **x**, and operator $\mathbf{C} : \Omega \rightarrow [0;1]^n$, defined as $\mathbf{x^\wedge} := \mathbf{Cx}$ will be called a cumulating operator. In addition we will distinguish two special structures in set Ω:

$$\mathbf{e} := \left(\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n}\right), \quad \mathbf{s} := (0, 0, ..., 0, 1) \tag{1}$$

In our deliberations, the above vectors will play the roles of model structures, against which we will calculate measures of concentration of other structures. In order to simplify further pondering let us make an additional assumption that our data is on the individual object level (no aggregation). This assumption does not decrease the generality of the study, but makes it much easier.

If we denote the measure of dissimilarity (differentiation or distance, which does not need to be a metric) of two structures by d, the structure of owned goods (shares) by **x**, then indicator Ψ which valuates the concentration of goods, which distribution between shareholders is defined by structure **x**, can be defined by the following formulae:

$$\Psi(\mathbf{x}) := \frac{\mathrm{d}[\mathbf{C}(\mathbf{e}); \mathrm{C}(\mathbf{P}(\mathbf{x}))]}{\mathrm{d}(\mathbf{C}(\mathbf{e}); \mathbf{s})} \tag{2}$$

$$\Psi(\mathbf{x}) := \frac{\mathrm{d}(\mathbf{e}; \mathbf{P}(\mathbf{x}))}{\mathrm{d}(\mathbf{e}; \mathbf{s})} \tag{3}$$

where: **e, s** – defined by formula (1).

Naturally, not all measures of distance (dissimilarity) are equally "good" for constructing a coefficient of concentration. Most well known and widely used measure of distance (dissimilarity) is Minkowski's metric, which can be written as follows:

$$\mathrm{d}_p(\mathbf{x},\mathbf{y}) := \left[\sum_{i=1}^{n} \left|x_i - y_i\right|^p\right]^{\frac{1}{p}}, \quad 1 \le p \le \infty \tag{4}$$

where: $\mathbf{x} = (x_1, ..., x_n)$, $\mathbf{y} = (y_1, ..., y_n) \in \Re^n$.

Practitioners that study levels of differentiation of income as well as goods owned by elements of a set of objects, often present a list of postulates for indicator Ψ, used to measure concentration. Generally, meaning in the case when data is aggregated, it refers to dissimilarity between the structure of elements and structure of goods that those elements own. In the case of detailed data, the most important and widely accepted postulates are:

- indicator Ψ reaches its minimal value when goods are evenly distributed amongst all objects,
- value of the indicator is in line with the principle of transfers, which states that a transfer of any amount of good from a "poorer" object to a "richer" object always results in an increase of inequality,
- transfer sensitivity axiom states that the impact of a transfer of goods from a "poorer" object to a "richer" object on the value of the indicator, when the value of the transfer is constant, is proportional to the amount of goods owned by the "poorer" object,
- indicator Ψ reaches its maximal value when all goods are owned by a single object.

It seems worthwhile to add to the above four postulates another one which states that the values of indicator Ψ are normalized, meaning the codomain of indicator Ψ is equal to a closed interval [0, 1]. This addition allows for comparisons of values of coefficients of concentration of sets of objects with different cardinalities. Please note that neither of the two most popular indicators, that is Gini Index nor HHI, satisfy that postulate:

- Gini Index has a codomain of $\left[0, \frac{n-1}{n}\right]$
- HHI has a codomain of $\left[\frac{1}{n}, 1\right]$.

Because of that, both indicators are normalized for practical use:

$$\mathrm{HHI}^* := \frac{\mathrm{HHI}-\frac{1}{n}}{1-\frac{1}{n}}, \qquad \mathrm{Gini}^* = \frac{n}{n-1}\mathrm{Gini} \tag{5}$$

where: structure $\mathbf{x} = (x_1, x_2, ..., x_n) \in \Omega$

HHI, Gini – indicators calculated according to original formulae [Gini 1914, Glasser 1962, Hirschman 1964, Herfindahl 1955]:

$$\mathrm{HHI}(\mathbf{x}) = \sum_{i=1}^{n}(x_i)^2, \ \ \mathrm{Gini}(\mathbf{x}) := \frac{1}{n}\sum_{i=1}^{n}(2i-n-1)x_i^{'}, \ \ \mathbf{x}^{'} = \mathrm{P}\mathbf{x} = (x_1^{'}, x_2^{'}, ..., x_n^{'})$$

It can be easily shown that normalized indicators HHI* and Gini* satisfy all above conditions. Moreover, it is worth noting that if an indicator Ψ satisfies the above, and a function f : [0; 1] → [0; 1] is non-decreasing and $f(0) = 0$, $f(1) = 1$, then an indicator calculated as a composition of function f and indicator Ψ: $f(\Psi)$, will also satisfy the above postulates. It has been shown in the work [Binderman et al. 2013c] that when using Minkowski's metric, an indicator constructed by using formula (2) or (3) also satisfies the above postulates. Furthermore, it has been showed that the following equations hold true:

$$\mathrm{HHI}^* = \left[\frac{\mathrm{d}_2(\mathbf{e};\mathbf{P}(\mathbf{x}))}{\mathrm{d}_2(\mathbf{e};\mathbf{s})}\right]^2, \ \ \mathrm{Gini}^* = \frac{\mathrm{d}_1[\mathbf{C}(\mathbf{e});\mathbf{C}(\mathbf{P}(\mathbf{x}))]}{\mathrm{d}_1(\mathbf{C}(\mathbf{e});\mathbf{s})} \tag{6}$$

where: d_1, d_2 – metrics as defined by formula (4).

This means that by using Minkowski's metrics we can construct many different coefficients of concentration which is significant from a practical standpoint. It should be mentioned that using only one coefficient, even one that is universally acclaimed, can lead to temporarily not noticing even relatively large scale changes. In addition, different coefficients can point to different directions of changes in concentration. In order to showcase this situation, let us consider the following example which deals with changes on a market consisting of different subjects. Table 1 shows the changes in shares in a market of 10 fictitious subjects in a span of few years. For each of the time series points we have calculated the values of the two most popular, normalized coefficients of concentration HHI* and Gini* as well as a coefficient named Radar, which was proposed by the authors of this work in [Binderman et al. 2012]. In that work it was shown that the measure Radar satisfies all above postulates. The basis for constructing that coefficient is a radar chard of structural vectors [Binderman et al. 2008, 2009, 2013c, Binderman 2011, Binderman et al. 2013a, b].

Please mind that structural vectors denoted (in the first row) by symbols **s**1, **s**2, …, **s**6 (defined by coordinates in the below 10 rows) differ significantly between each other (Table 1). For example: **s**1 = (0.0105; 0.0111; 0.0112; 0.0114; 0.0115; 0.0116, 0.01395; 0.2933; 0.3099; 0.315) while **s**6 = (0.055; 0.055; 0.055; 0.055; 0.055; 0.055; 0.055; 0.055; 0.055; 0.505). In the bottom three rows we have given the values of coefficients of concentration for those structures, calculated via measures HHI*, Gini* and Radar, respectively. In both, the bottom three rows and Figure 1, the different reactions, in terms of changes of the level of values and the direction thereof, of individual coefficients are clearly visible.

Table 1. Illustration of values of coefficients of concentration for six exemplary structures (scenarios) – synthetic data

	S1	S2	S3	S4	S5	S6
o01	1.105%	3.0135%	1.000%	1.982526%	5.4995%	5.50%
o02	1.110%	3.0185%	1.996%	1.982526%	5.4995%	5.50%
o03	1.120%	3.0200%	3.000%	1.982526%	5.4995%	5.50%
o04	1.140%	3.0250%	4.000%	1.982526%	5.4995%	5.50%
o05	1.150%	3.0500%	6.000%	1.982526%	5.4995%	5.50%
o06	1.160%	3.0900%	7.000%	5.750000%	5.4995%	5.50%
o07	1.395%	3.1600%	7.900%	7.250000%	5.4995%	5.50%
o08	29.330%	5.0950%	8.600%	10.000000%	5.4995%	5.50%
o09	30.990%	33.0000%	11.001%	20.000000%	5.4995%	5.50%
o10	31.500%	40.5280%	49.503%	47.087373%	50.5045%	50.50%
HHI*	0.2025000	0.2025308	0.2025470	0.2025010	0.2025450	0.2025000
GINI*	0.6939333	0.6203567	0.6002911	0.6574722	0.4500500	0.4500000
Radar	0.7064599	0.7229833	0.7151906	0.7453993	0.6247594	0.6247059

Source: Own research.

In the case of transition from structure **s**1 to **s**6 we observe no change in the value of coefficient HHI* with a relatively large changes of both indicators Gini* and Radar. Moreover, it is worth noting that the value of coefficient HHI* for structures **s**1–**s**6 differs greatly from the values of both indicators Gini* and Radar. Additionally, the levels of change of values in Gini* and Radar differ greatly between each other when transitioning from **s**1 to **s**6. The difference is even more apparent in the transition between structures **s**1 and **s**2, where the direction of change in Gini* value is opposite to that of Radar. The changes can be viewed in detail in Figure 1. Please note that the range of values of HHI*

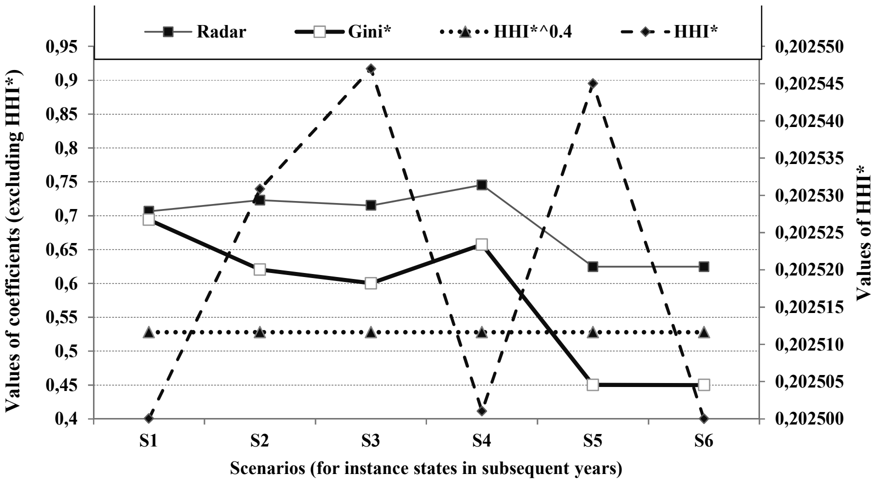

Fig. 1. Values of coefficients for structures defined in Table 1
Source: Own research.

coefficient of concentration in the above chart is different from that of Gini* and Radar, and is present on the right hand axis. It is necessary, because the range of values of HHI* is on a different level to that of Gini* and Radar, and the purpose of Figure 1 was to present the direction of changes in values of coefficients induced by changes in the underlying structure. In order to depict minute changes of value of coefficient HHI* in comparison to values of measures Gini* and Radar, according to the range of values present on the left hand axis, we have plotted values of an additional coefficient $(HHI^*)^{0.4}$.

In order to showcase the differences in measuring concentration via different coefficients even more, we will present charts of values of those coefficients in comparison to certain reference models. The simplest solution seems to be specifying structure models by using ordered family of Lorenz curves [Gastwirth 1971, Arnold 1987] and, based on them, constructing k-element structures of goods. Going back to the example from Table 1, we can construct structures of ten coordinates and identify them with, for example, structures of income which correspond to decile groups of workers in different countries or market shares of ten companies. Let us consider two families of curves:

$$L(t) = t^{\alpha}, \quad t \in [0;1], \ \alpha \geq 1 \tag{7}$$

$$L_{\mu}(t) = F_{\mu}\left(F_0^{-1}(t)\right), \ t \in [0;1], \ \mu \geq 0 \tag{8}$$

where: F_{μ} – cumulative distribution function for a normal distribution with mean μ and standard deviation equal to 1.

In Figure 2, we have presented two families of Lorenz curves and in Figure 3 charts of values of chosen coefficients of concentration which construction was based on Minkowski's metric. In order to create the structures we have used aggregation based on quantiles, choosing structures with 66 coordinates – which correspond to 66 subregions of Poland. To increase the clarity of presented charts we have utilized the following notation:

- m(*p; q*) – for the *q*-th power of coefficient as defined in formula (2), while using distance as defined in (4),
- M(*p; q*) – for the *q*-th power of coefficient as defined in formula (3), while using the distance as defined in formula (4).

Naturally, with those definitions we have HHI* = M(2; 2) and Gini* = m(1; 1). Analysis of Figures 2 and 3 can aid a researcher (analyst) in choosing a right coefficient as one can choose a coefficient of appropriate sensitivity based on the shape of the curve of concentration.

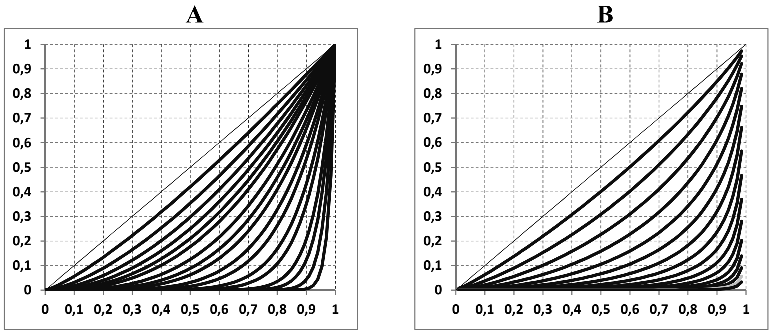

Fig. 2. Charts for curves of concentration defined by formulae (7) and (8)
Source: Own research.

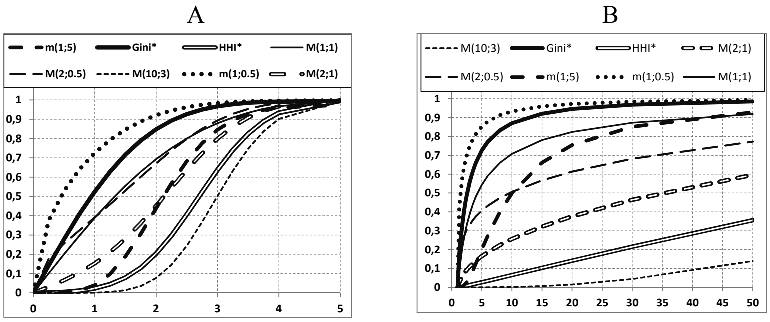

Fig. 3. Charts for chosen coefficients of concentration induced by curves of concentration presented in Figure 2
Source: Own research.

In Figure 2A we have presented charts for α = 1; 1.25; 1.5; 1.75; 2; 2.25; 2.5; 3; 3.5; 4; 5; 6; 8; 10; 15; 20; 30; 50. In Figure 2B we have presented charts for values $\mu = \mu_i = 0 + 0{,}25i$, where $i = 0,1,\ldots,16$ and μ = 5. Similarly, in Figure 3 we have presented charts for chosen coefficients of concentration for structures with 66 coordinates (corresponding a structure of a good for 66 objects) aquired from the curves of concentration from Figure 2.

CHANGES IN CONCENTRATION OF GROSS VALUE ADDED

Poland is divided into six regions which are further divided into 66 subregions [Central Statistical Office, The Nomenclature of Territorial Units for Statistical Purposes (NTS), www.stat.gov.pl]. Possessing data for a period of only 11 years, we do not expect to see major changes in the sectors of agriculture, forestry, fishing and hunting. However, even now certain trends can be seen. In this work, due to space constraints, we have decided to limit our inquiry to measure concentration with only the most popular indicators in their normalized forms: HHI* and Gini*. Based on them, we will try to assess the direction of changes in each of the six individual regions. Let us note that, if follows from Figures 1 and 2 that those coefficients have different sensitivities to changes of structures of shares in gross value added. In Figure 4 we have presented changes in concentration of value added during the period 2000–2010 in six geographical regions.

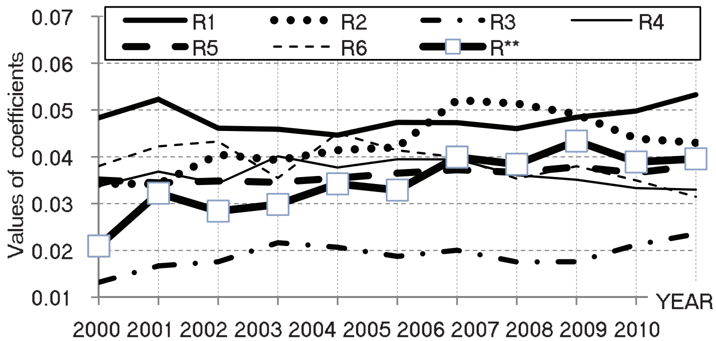

Fig. 4. Values of coefficients of concentration HHI* depicting concentration of GVA in sectors of agricutlure, forestry, fishing and hunting during 2000–2010
Source: Own research.

It can be easily seen that no clear drop in the level of concentration takes place, neither in the collectivity as a whole not in individual regions. The situation in those sectors is so unequivocal that the image of changes in a dynamic approach when using different coefficients of concentration is very similar, changing only in terms of the range of ordinate values. The chart on Figure 5, by depicting the values of measure HHI*, illustrates the concentration of gross value added in a collectivity of six regions as well as the individual regions themselves. The notation is as follows: R1 – central, R2 – south, R3 – east, R4 – north-west, R5 – south-west, R6 – north, R** – the collectivity of six regions. In Figure 5 we have presented values of coefficients of concentration M(2; 0.5), M(1; 1), Gini*, HHI* of gross value added in sectors of agriculture, forestry, fishing and hunting in the collectivity of 66 subregions. The values of HHI* use the right hand axis.

Based on Figure 5, we can conclude that changes in the considered sectors are occuring slowly and steadily. Because of that it is irrelevant which coefficient we use, we will always get the same trend of changes in its value. However, if we look at the value added for all sectors combined we can see some differences. In Figure 6 we have presented changes in concentration of GVA for all sectors combined for the collectivity of

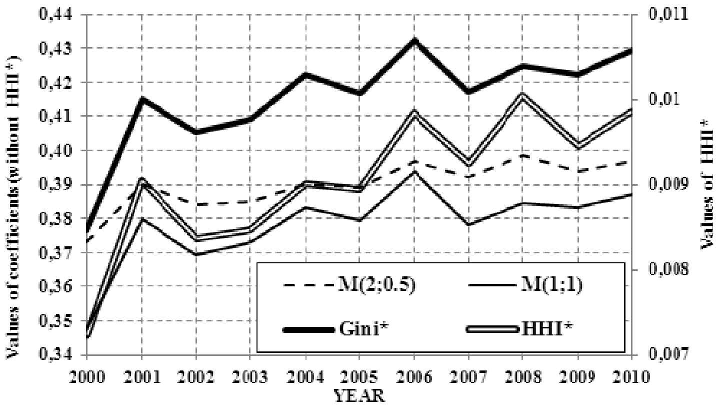

Fig. 5. Values of chosen coefficients of concentration of GVA in a collectivity of 66 subregions during 2000–2010

Source: Own research.

six regions measured by values of HHI* and chosen appropriately to the set of values coefficient m(p; q) of two variants: without capital citz Warsaw and with including it in the central region (denoted "z W-wa"). Because of different levels of values of individual coefficients the chart has been created with two vertical axes. Values of coefficients HHI*, m(1; 2.39), M(2; 1.669) and m(1; 3.33) are depicted on the right hand axis while values denoted "z W-wa" are on the left hand one. It is easily seen that coefficients of concentration HHI* and m(1; 2.39) without the capital show a reversed direction of changes between 2001 and 2002 as well as between 2008 and 2009. Those two coefficients when the capital city is included have the same direction of changes, but differ in the intensity of changes during 2006–2010. Figure 6 does not refute the thesis that after the access to the EU, if Warsaw is to be excluded, the concentration of value added in the collectivity

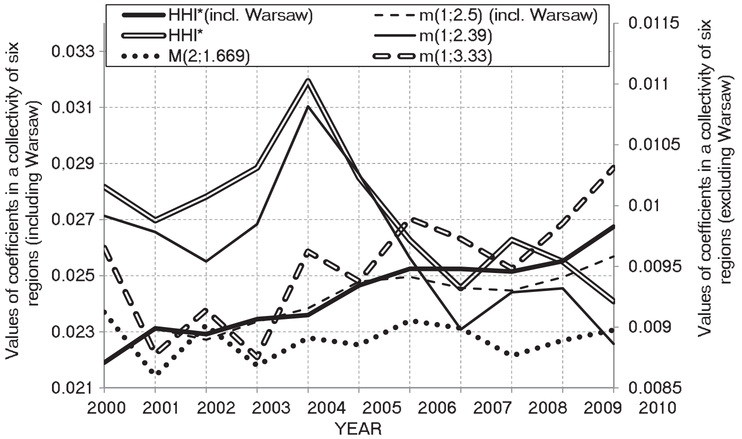

Fig. 6. Values of chosen coefficients of concentration of gross value added for all sectors combined in a collectivity of six regions, during 2000–2010

Source: Own research.

of six regions has a downward trend. When Warsaw is included the values of coefficients HHI* and m(1; 2.39) increase with a slight drop in years 2007 and 2008. In Figure 6 we have also included charts for values of coefficients M(2; 1.669) and m(1; 3.33) for a 65-element collectivity of subregions (Warsaw excluded). The former can be written as coefficient $(HHI^*)^{0.8245}$, while the latter as $(Gini^*)^{3.33}$. We have done this in order to increase the clarity of the chart. The chart of the former indicates that after a division into regions and excluding Warsaw concentration remains constant throughout the time period, while the chart of the latter an upward trend can be discerned.

SUMMARY

Based on the results presented in this work for the changes of concentration of GVA created in the sectors of agriculture, forestry, fishing and hunting during 2000–2010 we can see that in general there is no problem when choosing an indicator, because the changes of values of all shown measures of concentration in the discussed sectors are occurring slowly and steadily. Because of that it does not matter which coefficient we use as we will get a similar trend of changes of its value. We have confirmed that the concentration of GVA for this sector in a collectivity of 66 subregions does not decrease after Poland's access to the EU, but we can even see a gentle upward trend (Fig. 5). If we limit ourselves to only a measure of HHI* on a collectivity of six regions (Fig. 4) we can see a definitive upward trend. Naturally, within each region (collectivity of subregions that makes up a region) the situation is a bit different (Fig. 4). However, if we carefully analyze individual charts, that present changes in concentration of GVA created by all sectors combined within a collectivity of six regions, in Figure 6 we can clearly see that the changes in values of considered coefficients give different results of changes in concentration between individual years. This can lead to different conclusions. For example, when we exclude Warsaw and consider the collectivity of 65 subregions, then based on values of M(2; 1.669) (meaning $(HHI^*)^{0.8345}$) one can conclude that during 2000–2010, the level of concentration of GVA was constant apart from slight fluctuations. However, when analyzing values of m(1; 3.33) one can deduce that that an upward trend was present. On the other hand, if we utilize measures HHI* or m(1; 2.39) then we can find that during 2004–2007 concentration in the collectivity of six regions decreases in year 2008 and in 2009 a slight increase is visible, only to have the value drop in 2010 to a level below that of 2007. Thus, if we base our deliberations on the values of those indicators we will be certain that the hypothesis stating that EU subsidies decrease regional differentiation is true. This cannot be said in the case of sectors of agriculture, forestry, fishing and hunting. Let us note here that the synthetic example given in the beginning of this work as well as real data about GVA, clearly point that using only one indicator, even one universally acclaimed, can lead to not noticing occurring structural changes, even when those changes are relatively large. Moreover, before choosing means to measure concentration, it is worthwhile to ponder for a moment what is the character of the analyzed changes, as different indicators have different sensitivities and can, in boundary conditions, point to different directions of changes in concentration. It is important to chose a measure that is most sensitive to the aspect that the researcher is trying to analyze. Because of that it is advisable to create an analytical model that includes the analyzed aspect of the changes in order to choose the right indicator. One can use synthetic data to choose an indicator from those available in literature. Technology for creating measure, presented in this work, shows that a researcher has some leeway in constructing a new measure or transforming one already available.

REFERENCES

Główny Urząd Statystyczny (GUS). Produkt krajowy brutto i wartość dodana brutto według województw i podregionów w latach 2000–2010. http://www.stat.gov.pl/ gus/5840_13605_PLK_HTML.htm (6.12.2012).

Arnold, B.C. (1987). Majorization and the Lorenz Order: A Brief Introduction, Lecture Notes in Statistics 43, Springer-Verlag, Berlin

Ávila, F., Flores, E., López-Gallo, F., Márquez, J. (2013). Concentration indicators: Assessing the gap between aggregate and detailed data. IFC Bulletin, 36. Statistical issues and activities in a changing environment, 542–559.

Barnett, R.A., et al. (2005). College Mathematics for Business, Economics, Life Sciences, and Social Sciences, 10th ed. Prentice-Hall, Upper Saddle River.

Binderman, Z. (2011). Matematyczne aspekty metod radarowych. MIBE, XII, 2, 69–79.

Binderman, Z., Borkowski, B., Szczesny, W. (2008). O pewnej metodzie porządkowania obiektów na przykładzie regionalnego zróżnicowania rolnictwa. Metody Ilościowe w Badaniach Ekonomicznych, IX, 39–48.

Binderman, Z., Borkowski, B., Szczesny, W. (2012). Radar coefficient of concentration. Quantitative Methods in Economics, XIII, 2, 7–21.

Binderman, Z., Borkowski, B., Szczesny, W. (2013). Zastosowanie metryki Minkowskiego do pomiaru zmian koncentracji Quantitative Methods in Economics, XIV, 3, 7–21.

Binderman, Z., Szczesny, W. (2009). Arrange methods of tradesmen of software with a help of graphic representations CASTR. Wyd. WSFiZ, Siedlce, 117–131.

Binderman, Z., Borkowski, B., Prokopenya, A., Szczesny, W. (2013). Mechanisms of Construction of the Radar Coefficients of Concentration, Computer Algebra Systems in Teaching and Research. Collegium Mazovia, Siedlce, 7–15.

Binderman, Z., Borkowski, B., Prokopenya, A. Szczesny, W. (2013a). Radar Coefficients of Concentration, Verifications of Properties, CASTR. Collegium Mazovia, Siedlce, 16–28.

Calkins, S. (1983). The New Merger Guidelines and the Herfindahl-Hirschman Index. California Law Review, 71(2), 402–429.

Djolov, G. The Herfindahl-Hirschman: Index as a decision tool to business concentration in the banking sector: a general perspective. https://umsbe.wufoo.com/cabinet /276/298/53/p171_update3.docx (14.05.2013).

Gastwirth, J.L. (1971). A general definition of the Lorenz curve. Econometrica, 39, 1037–1039.

Gini, C. (1914). Sulla misura della concentrazione e della variabilità dei caratteri. Atti del R. Istituto Veneto di Scienze Lettere. Arti, Venezia.

Glasser, G.J. (1962). Variance formulas for the mean difference and coefficient of concentration. J. Am. Statistic. Assoc., 57, 648–654.

Herfindahl, O.C. (1955). Comment on Rosenbluth's measures of concentration. Business concentration and price policy. Ed. G. Stigler. Princeton University Press, Princeton.

Hirschman, A.O. (1964). The paternity of an index, AE Rev., 54 (5), 761–762.

Hoffmann, L.D., Bradley, G.L. (2007). Calculus for Business, Economics, and the Social and Life Sciences. 9th ed., McGraw Hill, New York.

Kwoka, J.E. (1985). The Herfindahl index in theory and practice, Antitr. Bull., 30, 915–947.

Lijesen, M.G. (2004). Adjusting the Herfindahl index for close substitutes: an application to pricing in civil aviation. Transportation Research Part E 40, 123–134
Matsumoto, A., Merlone, U., Szidarovszky, F. (2012). Some notes on applying the Herfindahl--Hirschman index A. Economics Letters. Taylor and Francis Journals, 19(2), 181–184.

ZASTOSOWANIE METRYKI MINKOWSKIEGO DO POMIARU ZMIAN KONCENTRACJI WARTOŚCI DODANEJ W SEKTORACH ROLNICTWA, LEŚNICTWA, ŁOWIECTWA I RYBACTWA

Streszczenie. Praca jest bezpośrednią kontynuacją pracy autorów dotyczącej konstrukcji nowych wskaźników koncentracji, przy użyciu metryki Minkowskiego ρ^p $(1 \leq p \leq \infty)$. W niniejszym artykule podano zastosowanie tych wskaźników w sektorze rolnictwa, leśnictwa, łowiectwa i rybactwa. Zbadano jak przebiegają zmiany koncentracji wartości dodanej wypracowanej w tym sektorze, dokonując porównań z innymi sektorami.

Słowa kluczowe: koncentracja rolnictwa, wartość dodana, współczynnik koncentracji, współczynnik Giniego, współczynnik Herfindahla-Hirschmana, metryka Minkowskiego

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